Deterministic Geometric Derivation of the Proton Effective Radius and Emergence of the Gravitational Constant Without Free Parameters

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Abstract

We present a deterministic geometric framework that derives the effective radius of the proton from first principles without free parameters. Starting from the sole condition that a stable confined wave must contain an exact integer number of wavelengths — a purely geometric requirement with no quantum mechanical input — we obtain $R_p = 4\hbar/(m_p c) = 0.841236$ fm, in agreement with CODATA 2022 within 0.058% and significantly more precise than current lattice QCD predictions of $0.820(14)$ fm.

The same geometric condition applied to any confined mass M yields a universal radius $R_{VED} = n\hbar/(Mc)$ whose product with the Schwarzschild radius is mass-independent across 74 orders of magnitude: $R_{VED} \cdot R_{Sch} = 4L_P^2$. This result allows the gravitational constant G to be derived geometrically without free parameters, establishing it as an emergent property of spacetime confinement at $n = 2$.

A direct algebraic consequence is $\Lambda = 0$: if G emerges complete from geometry, no geometric mechanism exists to generate an additional cosmological constant term. The observed cosmological redshift is reinterpreted as the projection of the temporal arrow onto distance ($dt \to -dz$), not as spatial expansion. The current standard cosmological model rests on a conceptual error — treating space and time as independent in the presence of velocity — that this framework resolves.

Keywords: proton radius, geometric confinement, gravitational constant, cosmological constant, deterministic physics, Einstein-VED

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1. The Problem

The effective radius of the proton has been one of the most persistent precision puzzles in modern physics. For decades, two families of measurements produced irreconcilable values:

- Electronic hydrogen spectroscopy and electron-proton scattering: $r_p \approx 0.877$ fm
- Muonic hydrogen spectroscopy: $r_p \approx 0.841$ fm

The discrepancy reached 7 sigma — the highest threshold of evidence in physics. CODATA 2022 has since converged toward $0.8414(19)$ fm, effectively vindicating the muonic measurements. But the resolution of the experimental puzzle has revealed a deeper theoretical problem: the best first-principles calculations from lattice QCD produce $0.820(14)$ fm, which does not agree with the experimental value within its own uncertainties.

This is not a technical detail. It is evidence that the current theoretical framework does not capture all the relevant physics. **The experiment has outrun the theory.**

We take this discrepancy seriously — as Einstein would have — and ask a more fundamental question: what is matter, really?

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2. The Question — What Is Matter, Really?

Quantum mechanics describes the behavior of matter with extraordinary predictive precision. But it does not answer the most fundamental question: what IS matter?

The Standard Model classifies particles and assigns quantum numbers and parameters. But those parameters — masses, charges, radii — are measured experimentally and inserted into the theory. They are not derived. The proton radius is one of them.

We follow the path that Einstein never abandoned: seeking a geometric and deterministic description of physical reality, where particle properties emerge from first principles without free parameters.

We start from the simplest possible observation:

Matter and energy are the same phenomenon in different states.

- **Matter** = wave confined within a closed boundary
- **Energy** = free wave, without confinement

They are not two distinct entities. They are the same spacetime vibrating in different regimes. The content does not differ substantially from the container.

This is not a new hypothesis — it is the most direct reading of the wave-particle duality that physics has known since De Broglie. It is surprising that with this simplicity and determination the question that changes everything had not been formulated before:

What geometric condition must a boundary satisfy for the confined wave to be stable?

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3. The Geometric Condition

The answer is simple, inevitable, and has nothing to do with quantum mechanics.

A wave train confined within a closed boundary is stable if and only if the boundary contains an exact integer number of wavelengths:

$$2\pi R = n\lambda, \quad n \in \mathbb{N}^+$$

The reason is purely geometric. If the number of cycles is not exactly integer, the wave does not close precisely — it accumulates a phase shift with each revolution. That phase shift is cumulative and destroys confinement. There is no possible exception. There is no interpretation. It is a topological necessity.

This condition does not require postulating quantization — quantization **emerges** from it. Integer numbers are not a mysterious property of quantum nature. They are the only geometric solution to the problem of stable confinement.

A crucial observation: the boundary does not have to be a circle. It can be any closed curve in spacetime — regular, irregular, complex. The only requirement is that it be topologically isomorphic to a circle and contain exactly $n\lambda$. Topology is the correct mathematical tool precisely because it studies properties that are preserved under continuous transformations, independently of the concrete shape of the boundary in Euclidean space. Affine spaces allow working with these geometries with full rigor before knowing all the details of the three-dimensional projection.

The effective radius of that isomorphism is:

$$R_{VED} = \frac{n\lambda}{2\pi} = \frac{n\hbar}{Mc}$$

where $\lambda = h/(Mc)$ is the Compton wavelength associated with mass M.

This radius is not the observable radius of a sphere in three dimensions. It is the effective radius of the topological boundary in 4D spacetime — the projection of a four-dimensional structure onto a temporal section $t = t_0$. When we measure the proton radius experimentally, we are measuring exactly that projection.

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4. The Result — Effective Radius of the Proton

The proton exists in a four-dimensional spacetime — three spatial and one temporal. Its confined wave not only closes in the three spatial dimensions but is anchored throughout the temporal line: it exists for all $t \in T$.

The stability condition requires the wave to close in all four dimensions. The minimum number of cycles for stable confinement in 4D is:

$$n = 4$$

This is not a postulate. It is the number of spacetime dimensions. Each dimension contributes exactly one cycle to confinement.

Applying $R_{VED} = n\hbar/(Mc)$ to the proton with $n = 4$:

$$R_p = \frac{4\hbar}{m_p c} = \frac{2}{\pi} \cdot \frac{h}{m_p c} = \frac{2}{\pi}\lambda_p$$

Numerical calculation with CODATA 2022 values:

$$\lambda_p = \frac{h}{m_p c} = \frac{6.62607015 \times 10^{-34}}{1.67262192369 \times 10^{-27} \times 2.99792458 \times 10^{8}} = 1.321409 \times 10^{-15}\ \mathrm{m}$$

$$R_p = \frac{2}{\pi} \times 1.321409 \times 10^{-15} = 0.841236\ \mathrm{fm}$$

Comparison:

Source	Value	Method
Einstein-VED	0.841236 fm	Geometric, no free parameters
CODATA 2022	$0.8414(19)$ fm	Experimental
Lattice QCD	$0.820(14)$ fm	Standard Model

The geometric prediction agrees with CODATA 2022 with a discrepancy of 0.058% — within the experimental margin. The experimental uncertainty of CODATA 2022 is ± 0.019 fm. The Einstein-VED geometric prediction is an exact value — 0.841236 fm — with no intrinsic uncertainty. The only source of imprecision comes from how precisely we know the proton mass — which is also experimental data.

The Standard Model, with all its mathematical machinery, produces a value of $0.820(14)$ fm whose uncertainty is almost ten times larger than the experimental one and whose central value does not agree with observation. **The experiment has outrun the theory. Geometry reaches it.**

This prediction is not a fit to the current CODATA value. It is a direct algebraic consequence of $n = 4$ and the proton mass. As experimental measurements of the effective proton radius gain precision — as will inevitably happen with upcoming generations of muonic spectroscopy and elastic scattering experiments — the Einstein-VED prediction does not need to be revised. It is the value toward which future experiments should converge.

A deterministic theory does not fit the data — it predicts them. And the target is already marked: $R_p = 0.841236$ fm.

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5. The Universal Geometric Constant

We have established that $R_{VED} = n\hbar/(Mc)$ describes the effective radius of any confined mass, regardless of its nature or scale. With this foundation, we ask: does a deeper relationship exist between R_{VED} and the geometry of spacetime at large scales?

The first intuition was natural: if R_{VED} describes the internal geometry of matter, perhaps it bears a relationship to the Schwarzschild radius $R_{Sch} = 2GM/c^2$, which describes its external geometry. But direct comparison reveals an immediate asymmetry:

$$R_{VED} = \frac{n\hbar}{Mc} \quad \text{decreases with mass}$$

$$R_{Sch} = \frac{2GM}{c^2} \quad \text{increases with mass}$$

Equating the radii would be an error. They are distinct geometries living in distinct frameworks. But precisely that asymmetry suggests something deeper.

Nothing prevents using the same mass M in both frameworks and asking what mass each geometry describes:

From R_{VED}:

$$M_{VED} = \frac{n\hbar}{R_{VED} \cdot c}$$

From R_{Sch}:

$$M_{Sch} = \frac{R_{Sch} \cdot c^2}{2G}$$

These are two descriptions of the same physical mass. Requiring consistency imposes:

$$M_{VED} = M_{Sch} \implies \frac{n\hbar}{R_{VED} \cdot c} = \frac{R_{Sch} \cdot c^2}{2G}$$

The mass cancels algebraically. We obtain:

$$G(n) = \frac{R_{VED} \cdot R_{Sch} \cdot c^3}{2n\hbar}$$

This expression has a revealing structure: the left side depends on n and not on G; the right side depends on G and not on n. The equality allows **deriving G as an emergent constant**. We see that $G(n) = G_{universal}$ only for $n = 2$.

The deepest consequence is that **the product $R_{VED} \cdot R_{Sch}$ does not depend on mass**:

$$R_{VED} \cdot R_{Sch} = \frac{2n\hbar G}{c^3} = 4L_P^2 = 1.0449 \times 10^{-69}\ \mathrm{m}^2$$

Verification across 74 orders of magnitude in mass:

Object	Mass (kg)	R_{VED} (m)	R_{Sch} (m)	Product (m²)
Electron	9.11×10^{-31}	7.72×10^{-13}	1.35×10^{-57}	1.0449×10^{-69} ✓
Proton	1.67×10^{-27}	4.21×10^{-16}	2.48×10^{-54}	1.0449×10^{-69} ✓
Pen (10g)	1.0×10^{-2}	7.04×10^{-41}	1.49×10^{-29}	1.0449×10^{-69} ✓
Sun	1.99×10^{30}	3.54×10^{-73}	2.95×10^{3}	1.0449×10^{-69} ✓
Black hole $10^{10} M_\odot$	1.99×10^{40}	3.54×10^{-83}	2.95×10^{13}	1.0449×10^{-69} ✓

The mass is the window for accessing this constant — but the constant existed before any mass existed.

Constants as artifacts of units

Imagine measuring the volume of a room with width in inches, length in nautical miles, and height in cubits. Conversion factors would be needed to obtain a consistent result — constants that are not properties of the room but artifacts of our incompatible units.

That is exactly what happens with c, $\hbar$ and G:

- The standard definition of the meter uses the second as reference. If we used the "light-second" as a spatial unit, $c = 1$ — it would vanish as a constant.
- If we used the matter wavelength as a unit of mass, $\hbar = 1$.
- If we used the product $R_{VED} \cdot R_{Sch}$ as a unit of area, $G = 1$.

c, $\hbar$ and G are not fundamental properties of the universe. They are conversion factors between units that humans chose historically without knowing they were measuring the same reality from different angles.

The only real constant is:

$$\boxed{P = R_{VED} \cdot R_{Sch} = 4L_P^2 = 1.0449 \times 10^{-69}\ \mathrm{m}^2}$$

A surface. Two dimensions. The minimal geometric imprint of reality.

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6. The Algebraic Trap — $\Lambda = 0$ and the Conceptual Error of Standard Cosmology

If G emerges from pure geometry for $n = 2$, Einstein's field equations acquire a completely new meaning. The right-hand side of the equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

no longer contains an empirically measured constant inserted without explanation. It contains a **derived**, **exact** and **geometric** constant.

This has an immediate and inevitable algebraic consequence for the cosmological constant Λ.

Einstein wrote the equations without Λ in 1915. In 1917 he introduced it to maintain a static universe — and later called it his greatest mistake. In 1998, following Type Ia supernova observations, the scientific community reintroduced it to explain accelerated expansion, fitting its value to data without deriving it from any geometric principle.

The question that was never rigorously asked is: **does any geometric mechanism exist in 3S+T spacetime that generates a Λ term?**

The answer is no. The proof is algebraic:

G emerges complete and exact for $n = 2$. The product $R_{VED} \cdot R_{Sch} = 4L_P^2$ is the only geometric constant of the framework. No value of n — nor any combination of radii — generates an additional term with the dimensions of Λ (length^{-2}) without introducing a new free parameter.

Therefore:

$$\boxed{\Lambda = 0}$$

Not as a hypothesis. Not as aesthetic preference. As a direct algebraic result of G being pure geometry.

Einstein's original 1915 equations were correct. The trap was set twice:

First time — 1917: Einstein himself introduced Λ for physically incorrect reasons. He acknowledged this himself.

Second time — 1998: The community reintroduced Λ as "dark energy" to fit accelerated expansion observations, without deriving it, without geometric justification, converting Einstein's self-confessed greatest mistake into a pillar of the standard cosmological model.

The consequences of $\Lambda = 0$ are direct:

- Dark energy does not exist — it is an artifact of $\Lambda \neq 0$
- The 120-order-of-magnitude discrepancy between the observed value of Λ and that predicted by quantum field theory is resolved: $\Lambda = 0$ in both frameworks
- The ΛCDM model is built on a constant with no physical origin

The temporal arrow misidentified as spatial expansion

For those who defend accelerated expansion as irrefutable evidence for $\Lambda \neq 0$, we pose an elementary question:

In physics, time flows in one direction: $dt \geq 0$. The cosmological redshift is interpreted as spatial expansion. But a simpler, geometrically consistent interpretation exists:

$$dt \to -dz$$

The temporal arrow projected onto distance. The further we observe, the further back in time we look. The redshift is not spatial expansion — it is the signature that we are accessing progressively older instants t_0 as distance z increases.

This observation is not minor. It has profound conceptual consequences. From special relativity we know that $v = ds/dt$ expresses the degree of dependence between space and time:

- For $v = 0$: space and time are independent and orthogonal
- For $v \neq 0$: space and time are dependent — not orthogonal
- For $v = c$: the dependence is total — $ds = dt$ in natural units

Whenever a velocity exists, space and time cease to be independent. Treating them as separate and independent dimensions in the presence of motion is a serious conceptual error — not an acceptable approximation but a contradiction with the very foundations of relativity.

Cosmological redshift involves velocities. Separating dz from dt as if they were orthogonal and independent dimensions — and attributing the redshift exclusively to spatial expansion — is exactly that error.

$\Lambda \neq 0$ is the constant born from confusing the temporal arrow with spatial expansion. Its value is zero because the conceptual error that generated it does not exist in the real geometry of the universe.

The current cosmological model does not have a minor error to correct. It has a structural error at its foundations.

Einstein was right twice: when he wrote the equations without Λ, and when he called Λ his greatest mistake.

References

1. Estrada Díaz, V. (2026). *Einstein-VED: The theory that will reconcile Quantum Mechanics with Einstein's Relativity* (v17). Zenodo. DOI: 10.5281/zenodo.18816405

2. Estrada Díaz, V. (2026). *ES_04: Deterministic derivation of the proton radius and R_VED formula*. Zenodo. DOI: 10.5281/zenodo.18637554

3. Estrada Díaz, V. (2026). *ES_07: Gravity as a geometric relation of radii*. Zenodo. DOI: 10.5281/zenodo.18637554

4. Estrada Díaz, V. (2026). *ES_08: Planck constant as a geometric relation of radii*. Zenodo. DOI: 10.5281/zenodo.18637554

5. Estrada Díaz, V. (2026). *ES_11: Fundamental geometry of the universe: ++++ metric, dual universe and geometric origin of G and ħ*. Zenodo. DOI: 10.5281/zenodo.18637554

6. Estrada Díaz, V. (2023-2026). *Galactic dynamics analysis with Gaia data: refutation of dark matter*. GitHub. https://github.com/quark-cha/materia_oscura

7. Estrada Díaz, V. (2000-2026). *Complete theoretical framework Einstein-VED*. https://estradad.es/teorias

8. CODATA 2022. *Recommended Values of the Fundamental Physical Constants*. NIST. https://physics.nist.gov/cuu/Constants

9. Xiangdong Ji et al. (2023). *Lattice QCD calculation of the proton charge radius*. Physical Review Letters.

10. Antognini, A. et al. (2013). *Proton structure from the measurement of 2S-2P transition frequencies of muonic hydrogen*. Science, 339, 417-420.

11. Gaia Collaboration (2022). *Gaia Data Release 3*. Astronomy & Astrophysics, 649, A1.

12. Einstein, A. (1915). *Die Feldgleichungen der Gravitation*. Sitzungsberichte der Preussischen Akademie der Wissenschaften.

13. Einstein, A. (1917). *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie*. Sitzungsberichte der Preussischen Akademie der Wissenschaften.

14. Riess, A.G. et al. (1998). *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*. Astronomical Journal, 116, 1009.

15. Particle Data Group (2024). *Review of Particle Physics*. Progress of Theoretical and Experimental Physics.

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