EN_35: The Emergence of Matter from Area Eigenvalues

Correspondence Between U(Sch) and U(H) and the Geometric Origin of Masses

Author: Víctor Estrada Díaz
Framework: Einstein-VED
DOI: 10.5281/zenodo.17172925
License: CC BY-NC-SA 4.0
Web: https://estradad.es

Abstract

This document establishes an explicit mathematical correspondence between two fundamental geometries:

- **U(Sch)**: the geometry associated with the Schwarzschild radius and the area eigenvalues of a black hole (Bekenstein-Hawking).
- **U(H)**: the two-dimensional holographic geometry (the H(2) source), where the closure structure S^1 that defines confined waves (matter) resides.

The connection between both geometries is the **geometric constant ñ = R_VED · R_Sch**, which acts as an invariant mapping between the radii of both geometries.

As a result, **the area eigenvalues $A_m \sim m$ (with $m \in \mathbb{N}$) transform into a mass series $M_m \sim \sqrt{m}$** in the quantum world, corresponding to stable states of confined waves on S^1.

This result suggests that **natural numbers are the fundamental building blocks of matter**, and that the stability condition (an integer number of wavelengths on the contour) acts as a filter that selects which values of m produce stable particles in our universe.

Table of Contents

1. Defined Geometries
2. The Fundamental Geometric Constant ñ
3. Spectrum in U(Sch): Area Eigenvalues
4. Mapping to U(H) via ñ
5. Closure Geometry S^1 in U(H)
6. Relation Between Radius and Contour Length
7. Wave Interpretation of Mass
8. Complete Mapping Result
9. The Mass Series $M_m \sim \sqrt{m}$
10. The Emergence of Matter from Natural Numbers
11. The Stability Condition as a Filter
12. Physical Implications
13. Open Questions and Future Research Lines
14. Conclusion
15. References

1. Defined Geometries

Two fundamental geometries are considered within the Einstein-VED framework:

- **U(Sch)**: geometry associated with the Schwarzschild radius and the area eigenvalues of a black hole (Bekenstein-Hawking). This is the geometry of classical gravity and black hole thermodynamics.
- **U(H)**: two-dimensional holographic geometry (the H(2) source, see EN_0 and EN_11), where the closure structure S^1 that defines wave confinement resides. This is the geometry of information and holography.

Both geometries are manifestations of the same underlying reality, and their connection is established through a universal constant.

---

2. The Fundamental Geometric Constant ñ

The geometric constant ñ is defined as the product of the two fundamental radii of the Einstein-VED framework:

$$ñ = R_{\text{VED}} \cdot R_{\text{Sch}}$$

where:

- **R_VED**: holographic radius (wave confinement)

$$R_{\text{VED}} = \frac{nh}{Mc}$$

- **R_Sch**: Schwarzschild radius (gravitational horizon)

$$R_{\text{Sch}} = \frac{2GM}{c^2}$$

Substituting the expressions:

$$\tilde{n} = \left(\frac{nh}{Mc}\right) \cdot \left(\frac{2GM}{c^2}\right) = \frac{2nhG}{c^3}$$

Key observations:

1. **The mass M cancels exactly.** Therefore, ñ is a universal constant, independent of mass.
2. For universal gravity, n = 2, we obtain:

$$\tilde{n}_0 = \frac{4hG}{c^3} = 8\pi \ell_P^2$$

   where ℓ_p is the Planck length.

3. ñ has dimensions of area (L^2).

Interpretation: ñ is the **geometric coupling constant** that connects the two geometries. It acts as an invariant mapping:

$$\tilde{n}: U(Sch) \longrightarrow U(H)$$

---

3. Spectrum in U(Sch): Area Eigenvalues

In the geometry U(Sch), there exists a set of area eigenvalues (Bekenstein 1973, Hawking 1975):

$$A_m, \quad m \in \tilde{n}$$

Area quantization is assumed in units of ℓ_p^2:

$$A_m = m \cdot \Delta A$$

where ΔA is the fundamental area quantum. In loop quantum gravity (LQG), typically $\Delta A \sim 4\,\ell_p^2 \ln 2$, but the exact factor is not relevant for the qualitative structure.

Each eigenvalue A_m defines an associated Schwarzschild radius:

$$R_{\mathrm{Sch},m} = \frac{\sqrt{A_m}}{4\pi} = \sqrt{\frac{m\Delta A}{4\pi}} \propto \sqrt{m}$$

---

4. Mapping to U(H) via ñ

The geometric constant ñ acts as an **invariant mapping** between the two geometries.

For each eigenvalue A_m in U(Sch), the corresponding holographic radius in U(H) is defined by:

$$R_{\mathrm{VED},m} = \frac{\tilde{n}}{R_{\mathrm{Sch},m}}$$

Justification: Since ñ = R_VED · R_Sch is constant (independent of mass), if we know $R_Sch,_m$ from the eigenvalue A_m, we can determine $R_VED,_m$ as its geometric complement.

Substituting $R_Sch,_m \sim \sqrt{m}$:

$$R_{\mathrm{VED},m} \propto \frac{1}{\sqrt{m}}$$

---

5. Closure Geometry S^1 in U(H)

In the holographic geometry U(H), the fundamental confinement structure is a **circle S^1**. This is the topological projection of the closed contour of the confined wave in 4D.

The stability condition for a confined wave on S^1 is:

$$L = n\lambda, \quad n \in \mathbb{Z}^+$$

where:

- L is the contour length (perimeter)
- λ is the wavelength associated with the particle
- n is the **number of cycles** (integer number of wavelengths on the contour)

Stability criterion:

- If $n \in \mathbb{Z}^+$, the wave closes on itself → **stable state** (confined matter)
- If $n \notin \mathbb{Z}^+$, there is an accumulating phase shift → **unstable** (no lasting confinement)

---

6. Relation Between Radius and Contour Length

For a circle S^1, the contour length (perimeter) is related to the radius by:

$$L = R_{\text{VED}} = 2\pi r$$

Therefore, for each eigenvalue A_m:

$$L_m = R_{\mathrm{VED},m} = 2\pi r_m \propto \frac{1}{\sqrt{m}}$$

---

7. Wave Interpretation of Mass

In the Einstein-VED framework, the difference between matter and energy is a matter of confinement:

- **Free waves** → energy (no closed contour)
- **Confined waves on S^1** → mass (closed contour with integer n)

Mass is inversely associated with the contour scale. From R_VED = n h / (M c), we solve for M:

$$M = \frac{nh}{cR_{\mathrm{VED}}}$$

This is the fundamental relation between mass and holographic radius. For a stable state, n is the integer number of cycles (for the proton, n = 4).

---

8. Complete Mapping Result

For each eigenvalue A_m in U(Sch):

1. Define $R_Sch,_m = \sqrt{(A_m / 4\pi)} \sim \sqrt{m}$
2. Transform to $R_VED,_m = \tilde{n} / R_Sch,_m \sim 1/\sqrt{m}$ via the constant $\tilde{n}$
3. This $R_VED,_m$ defines a scale in U(H) (the radius of the circle S^1)
4. The associated perimeter is $L_m = 2\pi\ R_VED,_m$
5. The stability condition in U(H) requires $L_m = n\ \lambda$ with $n \in \mathbb{Z}^+$

6. When satisfied, a **stable state of matter** is obtained with mass:

$$M_m = \frac{nh}{cR_{\mathrm{VED},m}} = \frac{nh}{c} \cdot \frac{R_{\mathrm{Sch},m}}{\tilde{n}}$$

---

9. The Mass Series $M_m \sim \sqrt{m}$

Since $R_{Sch,m} \sim \sqrt{m}$, the corresponding mass is:

$$M_m \propto \frac{nh}{c} \cdot \frac{\sqrt{m}}{\tilde{n}} \propto \sqrt{m}$$

The relation induced by the mapping is:

$$M_m \sim \sqrt{m}, \quad m \in \tilde{n}$$

where m is the area eigenvalue in U(Sch).

The Series in Explicit Form

m	$M_m \sim \sqrt{m}$ (arbitrary units)	Possible physical correspondence
1	1.000	Fundamental mass? Electron?
2	1.414	
3	1.732	
4	2.000	Proton? (with appropriate scale factor)
5	2.236	

m	$M_m \sim \sqrt{m}$ (arbitrary units)	Possible physical correspondence
6	2.449	
7	2.646	
8	2.828	
9	3.000	
16	4.000	
25	5.000	
36	6.000	
49	7.000	
64	8.000	
81	9.000	
100	10.000	
121	11.000	
137	11.704	Z boson mass? (91.2 GeV ~ 11.7 × 7.8 GeV)
144	12.000	
…	…	

Note: The series is not expected to be exact in absolute values (there are scale factors depending on ñ, ΔA, etc.). But **the √m structure** is a falsifiable prediction of the framework.

---

10. The Emergence of Matter from Natural Numbers

The Fundamental Discovery

The area eigenvalues A_m in the gravitational geometry U(Sch) —which Bekenstein and Hawking associated with black hole entropy— are not just a property of event horizons.

They are the seed of all matter in the universe.

Through the geometric constant ñ = R_VED · R_Sch, each eigenvalue A_m (associated with a natural number $m \in \mathbb{N}$) projects into the holographic geometry U(H) as a radius $R_VED,_m$. This radius defines a circle S^1 of perimeter L_m. If an integer number of wavelengths fits on that perimeter, the wave becomes confined and stabilizes. That is **matter**.

Natural Numbers as the Building Blocks of the Universe

The natural numbers m (area eigenvalues in U(Sch)) are the source of matter in U(H).

- Each m generates a possible matter state with mass $M_m \sim \sqrt{m}$

- There is no continuity in masses; there is an underlying discrete structure
- Natural numbers are the "bricks" of physical reality

---

11. The Stability Condition as a Filter

The stability condition in U(H) acts as a **filter** on natural numbers. For an eigenvalue m to produce a stable particle in our universe, it must satisfy:

$$L_m = R_{\text{VED},m} = 2\pi r_m = n\lambda, \quad n \in \mathbb{Z}^+$$

This condition can be rewritten in terms of m and n:

$$n\lambda = \frac{\tilde{n}}{R_{\text{Sch},m}} = \frac{\tilde{n}}{\sqrt{m\Delta A/(4\pi)}} = \frac{2\sqrt{\pi}\,\tilde{n}}{\sqrt{m\Delta A}}$$

And since $\lambda \sim 1/M \sim 1/\sqrt{m}$ (from the mass relation), the condition is automatically satisfied for certain values of m and n.

The stability condition selects a subset of natural numbers as those that can give rise to stable particles. Not all m are equally probable.

---

12. Physical Implications

Implication	Description
Matter is projected geometry	What we call a "particle" is nothing more than an area eigenvalue in U(Sch) translated into a confined wave in U(H)
Natural numbers are fundamental	There is no continuity in masses; there is an underlying discrete structure
The constant ñ is the bridge	Connects gravity (black holes) with holography (wave confinement)
The Standard Model mass spectrum may be encoded	In the area eigenvalues of a black hole
The stability condition is the filter	Determines which values of m produce stable particles

Possible Correspondences with the Standard Model

With appropriate scale adjustment, some values of m could correspond to known particles:

- **m = 4**: Proton (mass 938 MeV) or neutron (940 MeV)
- **m = 1**: Electron (511 keV) or neutrino (very small mass)

- **m = 137**: Z boson (91.2 GeV) — curiously, 137 is the fine-structure constant
- **m = 81**: W boson (80.4 GeV)
- **m = 9**: Muon mass? (105.7 MeV, which is ~207 × electron mass, and 207 is close to 144, not 9; the relation is not direct but there may be additional factors)

It is important to note that the series $M_m \sim \sqrt{m}$ implies that masses grow as the square root of m. This means that to reach large masses (like the Z boson), large m (like 137) are needed.

---

13. Open Questions and Future Research Lines

Question	Description
What determines which values of m are "active"?	All natural numbers? Only primes? Only those that satisfy additional conditions?
How does m relate to Standard Model quantum numbers?	Is m a combination of quantum numbers such as spin, charge, flavor?
What role does the number 137 play?	It appears as the fine-structure constant and also as an area eigenvalue that could correspond to the Z boson.

Question	Description
What is the relationship between m and the 22 internal modes of the nucleon?	Could it be that m = 4 (proton) has 22 internal modes, and that this is a manifestation of the structure of m itself?
How does this series relate to the Standard Model mass hierarchy?	The mass hierarchy (electron, muon, tau, up/down quarks, etc.) could be encoded in different values of m.
Is there a connection with the Riemann Hypothesis?	Natural numbers and their properties (primes, distribution) are the subject of the Riemann Hypothesis. Could there be a deep connection between number theory and particle physics?

---

14. Conclusion

We have established an **explicit mathematical correspondence** between two fundamental geometries:

- U(Sch): area eigenvalues of a black hole
- U(H): confined waves on a circle S^1

The connection is the **geometric constant ñ = R_VED · R_Sch**.

The consequences are profound:

1. **The natural numbers $m \in \mathbb{N}$ are the fundamental building blocks of matter.** Each area eigenvalue in U(Sch) translates into a possible confined wave state in U(H) with mass $M_m \sim \sqrt{m}$.

2. **The stability condition (integer number of waves on the contour) acts as a filter** that selects which values of m produce stable particles in our universe.

3. **The Standard Model mass spectrum could be encoded in the area eigenvalues of a black hole.** This suggests a deep unity between quantum gravity and particle physics.

4. **The Einstein-VED framework not only solves existing problems but also opens entirely new questions** about the nature of numbers, the stability of matter, and the ultimate structure of the universe.

Geometry is the only law. Natural numbers are its alphabet. Particles are the words. And we, the observers, are the readers who, by reading, complete the work.

---

15. References

- Estrada Díaz, V. (2026). *EN_0: Einstein-VED - The Deterministic Framework That Completes Einstein's Program*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *EN_7: Gravity as a Geometric Relation of Radii*. Zenodo. 10.5281/zenodo.17172925

- Estrada Díaz, V. (2026). *EN_11: Fundamental Geometry of the Universe: ++++ Metric, Dual Universe, and Geometric Origin of G and h*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *EN_14: The 22 Internal Modes of the Nucleon*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *EN_30: Topological Foundations of the Einstein-VED Framework*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *EN_33: The Origin of Primordial Supermassive Black Holes*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *EN_34: Open Topics and New Research Directions*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *The Geometric Constant ñ and Its Relation to the Bekenstein-Hawking Law*. Zenodo. 10.5281/zenodo.17172925
- Bekenstein, J. D. (1973). *Black holes and entropy*. Physical Review D, 7(8), 2333.
- Hawking, S. W. (1975). *Particle creation by black holes*. Communications in Mathematical Physics, 43(3), 199-220.

End of EN_35