EN_4: Einstein-VED: Deterministic derivation of the proton and the R_VED formula

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The proton

1. Introduction

- We continue from the deterministic 3S+T framework.
- We have defined the container (spacetime) and the function f, ensuring causality and determinism.
- Quantum mechanics falls outside this analysis, since it cannot explain particle properties deterministically.

2. Particle with mass and intrinsic frequency

Every particle with mass m has an associated frequency ν and wavelength λ:

$$E = mc^2 = h\nu \quad \Rightarrow \quad \nu = \frac{mc^2}{h}$$

The associated Compton wavelength is:

$$\lambda_C = \frac{c}{\nu} = \frac{h}{mc}$$

This wavelength is geometric and deterministic, linked to 3S+T spacetime and the function f.

3. Geometric confinement axiom

For a wave associated with a particle to be confined within a perimeter or enclosure, the space must contain an exact number of complete waves:

$$L = n\lambda_C, \qquad n \in \mathbb{Z}^+$$

Otherwise, the enclosure will not be stable. Stability requires structural regularity along the timeline.

4. Proton confinement

The proton's minimum stable enclosure must close the wave in the three spatial dimensions and time, within the 3S+T geometric framework.

Phase closure condition

For a confined wave to close exactly along its own contour, the path length must contain an integer number of complete cycles:

$$L = n\lambda_C, \qquad n \in \mathbb{Z}^+$$

Each complete cycle entails a phase change of 2π radians. Therefore:

$$\Delta\phi = \frac{2\pi L}{\lambda_C} = 2\pi n$$

Exact closure requires:

$$\Delta\phi = 2\pi n, \qquad n \in \mathbb{Z}^{+}$$

If n is not an integer, the end of the wave does not match its beginning in phase, producing a residual phase shift:

$$\Delta\phi_{\text{res}} = 2\pi(n - \lfloor n \rfloor)$$

Stable confinement therefore requires an integer number of cycles along the contour.

Geometric closure in 3S+T

In Einstein–VED, minimum proton confinement comprises four closure planes:

1. XT plane: relation between X and T.
2. YT plane: relation between Y and T.
3. ZT plane: relation between Z and T.
4. XYZ plane: joint spatial closure.

The geometric condition specifies one complete cycle for each of these four closures.

Therefore, the **minimum number of cycles for stable confinement of the proton wave** is:

$$n_{\text{min}} = 4$$

This value does not arise from fitting the experimental radius, but from complete closure in the four planes of the Einstein–VED geometric construction.

The minimum perimeter is:

$$L_p = 4\lambda_p$$

The three closures XT, YT and ZT generate the three internal structures that, in this framework, geometrically explain the color property.

This establishes the confinement condition used in the following section to derive the proton radius from its mass.

5. Derivation of the proton mass and radius

The proton's intrinsic Compton wavelength is:

$$\lambda_p = \frac{h}{m_p c}$$

The minimum perimeter for stable confinement is:

$$L_p = n_{\min} \lambda_p = 4\lambda_p$$

The geometric relation between the perimeter and radius of spherical confinement gives:

$$2\pi r_p = L_p$$

Therefore:

$$r_p = \frac{L_p}{2\pi} = \frac{4\lambda_p}{2\pi} = \frac{2}{\pi}\lambda_p$$

Substituting the wavelength:

$$r_p = \frac{2h}{\pi m_p c}$$

Numerical calculation using CODATA 2022

- Planck constant:

$$h = 6.62607015 \times 10^{-34} \text{ J s}$$

- Proton mass:

$$m_p = 1.67262192595(52) \times 10^{-27} \text{ kg}$$

- Speed of light:

$$c = 2.99792458 \times 10^{8} \text{ m / s}$$

The resulting wavelength is:

$$\lambda_p = \frac{6.62607015 \times 10^{-34}}{1.67262192595 \times 10^{-27} \cdot 2.99792458 \times 10^{8}}$$

$$\lambda_p \approx 1.32140985 \times 10^{-15} \ m$$

And the radius:

$$r_p = \frac{2}{\pi}\lambda_p$$

$$r_p^{\text{VED}} \approx 0.84123564 \text{ fm}$$

Equivalent alternative formulation

$$r_p = n_{\min} \frac{h / (2\pi)}{m_p c} = 4\frac{h / (2\pi)}{m_p c}$$

The factor $h / (2\pi)$ arises from the geometric relation between perimeter and radius.

The formulation retains the Planck constant h and requires no additional constant.

5.1. Experimental comparison and QCD results

Reference date: 27 October 2026

Einstein–VED obtains the proton radius directly from its known mass, using the Einstein and Planck relations and the minimum number of cycles for stable wave confinement:

$$n_{\min} = 4$$

$$R_p = \frac{2h}{\pi m_p c} = \frac{2hc}{\pi E_p}$$

The experimental radius is neither an input parameter nor used to fit the result.

Comparison of results

Source	Radius ± uncertainty (fm)	Absolute difference (fm)	Relative difference
Einstein–VED, EN_4	**0.84123564021 ± 0.00000000026**	**0.00048564**	**0.0578%**
CODATA 2022	0.84075 ± 0.00064	0	0%
Lattice QCD, 2024	0.820 ± 0.014	0.02075	2.47%
Lattice QCD, 2026	0.860 ± 0.038 ± 0.023	0.01925	2.29%

Absolute and percentage differences are calculated relative to the CODATA 2022 experimental central value.

The uncertainty reported for Einstein–VED comes from propagating the experimental uncertainty in the proton mass. For QCD, the uncertainties published in the respective papers are reproduced.

Einstein–VED obtains a radius differing from the CODATA 2022 experimental central value by only 0.0578%, without fitting the formula to the observed radius.

In the two cited lattice QCD results, the differences from the CODATA central value are 2.47% and 2.29%, respectively.

The comparison directly illustrates the numerical proximity obtained by Einstein–VED through a closed geometric derivation.

The aim is to present a reproducible theoretical result and its agreement with available experimental data.

5.2. Calculation precision and future measurements

The Einstein–VED formulation calculates the proton radius from its mass or, equivalently, its rest energy:

$$R_p = \frac{2h}{\pi m_p c} = \frac{2hc}{\pi E_p}$$

In the International System, the Planck constant h and speed of light c have exact values.

The minimum confinement number $n = 4$ is established by the geometric condition for stable closure of the proton wave.

The propagated numerical uncertainty in the calculated radius therefore arises exclusively from our experimental knowledge of the proton mass:

$$\frac{\sigma_{R_p}}{R_p} = \frac{\sigma_{m_p}}{m_p}$$

The absolute uncertainty is obtained from:

$$\sigma_{R_p} = \left|\frac{\partial R_p}{\partial m_p}\right| \sigma_{m_p}$$

Since:

$$\frac{\partial R_p}{\partial m_p} = -\frac{2h}{\pi c m_p^2} = -\frac{R_p}{m_p}$$

It follows that:

$$\sigma_{R_p} = R_p \frac{\sigma_{m_p}}{m_p}$$

Using the mass uncertainty recommended by CODATA 2022:

$$\sigma_{m_p} = 0.00000000052 \times 10^{-27} \text{ kg}$$

The approximate result is:

$$\sigma_{R_p} = 2.6 \times 10^{-10} \text{ fm}$$

And the complete result:

$$R_p^{\text{VED}} = (0.84123564021 \pm 0.00000000026) \text{ fm}$$

Consequence of the mass–radius relation

Each experimental improvement in our knowledge of the proton mass, or its rest energy through $E_p = m_p c^2$, will allow the Einstein–VED calculated radius to be updated with a smaller propagated uncertainty.

The geometric expression remains unchanged:

$$R_p = \frac{2hc}{\pi E_p}$$

It requires no new parameters or recalibration of the formula using the experimental radius.

Thus the numerical precision of the calculated radius can improve as our experimental knowledge of the proton mass or rest energy advances.

6. Generalization: radius and mass associated with geometric confinement

The derivation of the proton radius establishes a general expression for all matter consisting of confined waves.

The intrinsic wavelength associated with a mass M is given by:

$$\lambda = \frac{h}{Mc}$$

Where:

- h = Planck constant.
- M = mass of the confined matter.
- c = speed of light.
- n = number of waves in the confinement.

The geometric closure condition gives:

$$L = n\lambda$$

For circular confinement:

$$2\pi R_{\text{VED}} = L$$

Therefore:

$$R_{\mathrm{VED}} = \frac{n\lambda}{2\pi}$$

Substituting the wavelength:

$$R_{\mathrm{VED}} = \frac{nh}{2\pi Mc}$$

This is the general formula for the Einstein–VED geometric radius.

Derivation of the mass

Solving for the mass:

$$M_{\mathrm{VED}} = \frac{nh}{2\pi c R_{\mathrm{VED}}}$$

And consequently:

$$M_{\mathrm{VED}}\, R_{\mathrm{VED}} = \frac{nh}{2\pi c}$$

For a given confinement number n, the geometric radius is inversely proportional to mass:

$$R_{\mathrm{VED}} \propto \frac{1}{M}$$

Within the Einstein–VED framework, this expression is proposed for all matter, from particles to macroscopic bodies, using the confinement number appropriate to each system.

R_{VED} characterizes the intrinsic geometry of wave confinement and should not necessarily be identified with observable body radii or the Schwarzschild gravitational radius.

Particular case of the proton

For the proton:

$$n_{\text{min}} = 4$$

$$R_{\text{VED}} = \frac{4h}{2\pi m_p c}$$

Simplifying:

$$R_{\text{VED}} = \frac{2h}{\pi m_p c}$$

Using the CODATA 2022 values employed in the derivation:

$$R_{\text{VED}} \approx 0.84123564 \text{ fm}$$

This recovers the result obtained by geometrically deriving the proton radius, without introducing new constants or fitting factors.

The formulation uses only the Planck constant h, retaining the confinement number n and geometric factor 2π explicitly.

7. Conclusion

1. Mass = a wave confined in 3S+T.
2. The radius and mass associated with geometric confinement (R_{VED} , M_{VED}) are exact and deterministic.
3. The formula $R_{\text{VED}} = \frac{nh}{2\pi Mc}$ will be a fundamental tool in subsequent developments.
4. For the proton, with $n = 4$, one obtains $r_p \approx 0.84123564$ fm by direct geometric derivation, without fitting the experimental radius.

5. The result differs from the CODATA 2022 experimental central value by 0.0578%, less than the differences of the two lattice QCD calculations considered.

6. The propagated numerical uncertainty in the Einstein–VED radius depends on the experimental uncertainty in our knowledge of the proton mass. Future improvements in determining its mass or rest energy will increase calculation precision without changing the geometric formula.

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