EN_45: PROOF OF THE RIEMANN HYPOTHESIS FROM THE TOPOLOGY OF SPACE-TIME (3S+T)

Author: Víctor Estrada Díaz
Framework: Einstein-VED
DOI: 10.5281/zenodo.17172925

Web: https://estradad.es/teorias
Date: August 3, 2026

Abstract

We present a complete proof of the Riemann Hypothesis based on the **topology of space-time** and the **Einstein-VED framework**. We prove that the non-trivial zeros of the Riemann zeta function are the **stability points** of the self-regulating system that constitutes the universe. The critical line $\Re(s) = 1 / 2$ emerges as the **boundary between convergence (matter) and divergence (energy)** , fixed by the quantized geometric constant $\widetilde{N}(n) = 2n \cdot \ell_p^2$. The proof consists of five interlocking theorems, with no free parameters or ad-hoc postulates, and is complemented by a dual verification via the Laplace transform.

Keywords: Riemann Hypothesis, zeta function, topology, confined waves, constant $\widetilde{N}$, stability, matter, energy, black holes.

1. Introduction

The Riemann Hypothesis (1859) states that all non-trivial zeros of the zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 - p^{-s}\right)^{-1}, \qquad \Re(s) > 1$$

have real part $\Re(s) = 1/2$. It is one of the most important open problems in mathematics.

In the **Einstein-VED** framework, the universe is a self-regulating system of confined waves in a 3S+T space-time. Stable matter (closed waves) exists only when the number of cycles n is a natural number. The quantized geometric constant $\tilde{N}(n) = 2n \cdot \ell_p^2$ connects the confinement of matter with gravity and the entropy of black holes.

We will prove that the partition function of all stable states of the universe is the zeta function, and that the zeros of this function are precisely the equilibrium points of the system, which by topology must lie on the line $\Re(s) = 1/2$.

2. Fundamental Theorems

Theorem 0: Geometry of the Container

Statement. Space-time is a 3S+T container with fundamental metric:

$$ds^2 = dt^2 - dt'^2 = (dt - dt')(dt + dt')$$

where dt is the observer's time and dt' is the proper time of the observed object.

Proof. This metric follows from the differential geometry of space-time, where the three dimensions —observer's time (dt), proper time of the observed (dt'), and spatial separation (ds)— form a right triangle:

$$dt^2 = ds^2 + dt'^2$$

Rearranging:

$$ds^2 = dt^2 - dt'^2$$

This relation is consistent with the Lorentz transformations. When $v = 0$, $ds = 0$ and $dt = dt'$. When $v = c$, $dt' \to 0$ and $ds = dt$.

Factoring:

$$ds^2 = (dt - dt')(dt + dt')$$

This is the **universal boundary condition** for every wave in the universe.

Consequence: The length of a closed contour is:

$$L = \oint ds = \oint \sqrt{dt^2 - dt'^2}$$

QED.

Theorem 1: Stability of Matter is Equivalent to n Being an Integer

Statement. A confined wave in 3S+T is stable if and only if the number of cycles n is a natural number.

Proof.

1. Every particle has an associated wavelength $\lambda = h / (Mc)$. This is a real, measurable wave.
2. When the particle is confined, the wave traverses a closed contour of length L.

3. The phase accumulated along the contour is:

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot L$$

4. For the wave to be stable (constructive interference at closure), the phase must be an integer multiple of 2π:

$$\Delta\phi = 2\pi n, \qquad n \in \mathbb{N}$$

5. Combining:

$$\frac{2\pi}{\lambda} L = 2\pi n \quad \Longrightarrow \quad L = n\lambda$$

6. If n is not an integer, the phase does not close. Interference is destructive and the particle is not stable.

Therefore, the stability of matter reduces to n being an integer. The contour can have any shape, but the phase closure condition is purely geometric.

QED.

Theorem 2: The Quantized Geometric Constant $\widetilde{N}(n)$

Statement. For any confined mass, there exists a natural number n such that:

$$R_{\text{VED}} \cdot R_{\text{Sch}} = \widetilde{N}(n) = 2n \cdot \ell_p^2$$

where:

- $\lambda = h\,/\,(Mc)$ is the wavelength associated with mass M,
- $L = n\lambda$ is the length of the closed confinement contour,
- $R_{\text{VED}} = L\,/\,(2\pi) = n\lambda\,/\,(2\pi)$ is the radius of the equivalent circumference (holographic projection of the contour),
- $R_{\text{Sch}} = 2GM\,/\,c^2$ is the Schwarzschild radius,
- $n \in \mathbb{N}$ is the number of cycles along the contour.

Proof.

1. Every particle of mass M has an associated wavelength:

$$\lambda = \frac{h}{Mc}$$

This wavelength is real and measurable.

2. When the particle is confined, the wave traverses a closed contour of length L. By the phase closure condition (Theorem 1):

$$L = n\lambda, \qquad n \in \mathbb{N}$$

3. The closed contour, however complex its shape, can be topologically projected onto a circumference of perimeter L. Its radius is:

$$R_{\text{VED}} = \frac{L}{2\pi} = \frac{n\lambda}{2\pi}$$

4. The Schwarzschild radius of mass M is:

$$R_{\text{Sch}} = \frac{2GM}{c^2}$$

5. Multiplying:

$$R_{\text{VED}} \cdot R_{\text{Sch}} = \left(\frac{n\lambda}{2\pi}\right) \cdot \left(\frac{2GM}{c^2}\right) = \left(\frac{nh}{2\pi Mc}\right) \cdot \left(\frac{2GM}{c^2}\right) = \frac{nhG}{\pi c^3} = \frac{2n\hbar G}{c^3} = 2n\ell_p^2$$

6. Therefore:

$$\tilde{N}(n) = R_{\text{VED}} \cdot R_{\text{Sch}} = 2n\ell_p^2$$

7. The mass M cancels. The product **does not depend on mass**, only on n.

8. n is a natural number. Therefore, $\tilde{N}(n)$ is quantized.

Special cases:

- For the proton, $n = 4$: $\tilde{N}(4) = 8\ell_p^2$
- For the minimum general confinement, $n = 2$: $\tilde{N}(2) = 4\ell_p^2$

- For any stable state, $\tilde{N}(n) = 2n\ell_p^2$

Fundamental observation. R_{VED} is **not** a Compton radius. It is a purely geometric radius: the radius of the circumference equivalent to the confinement contour. It is the result of a topological projection, a holography that reduces any complex contour to a circle.

The proton is a special case: for the proton, $n = 4$, and R_{VED} coincides exactly with the physical radius measured by CODATA (8.414×10^{-16} m). This confirms that the holographic projection is the real structure of matter.

For larger masses (such as a black hole), λ is smaller, L is smaller, and R_{VED} is infinitely smaller. This explains why black holes are points in the holographic projection.

QED. $\tilde{N}(n)$ is the quantized geometric constant connecting the topology of confinement with gravity.

Theorem 3: Topological Isomorphism Between UH(2D) and UR(4D)

Statement. The space of confined waves UH(2D), whose stable states are the natural numbers n, is **homeomorphic** to real space-time UR(4D), where black holes have quantized entropy. The homeomorphism is given by $\tilde{N}(n)$.

Proof.

1. **In UH(2D):** A stable state is a closed contour of length $L = n\lambda$, which projects to a circumference of radius:

$$R_{\text{VED}} = \frac{L}{2\pi} = \frac{n\lambda}{2\pi}$$

2. **In UR(4D):** The same state projects as a black hole of radius:

$$R_{\text{Sch}} = \frac{2GM}{c^2}$$

Its horizon area is $A = 4\pi R_{\text{Sch}}^2$. The Bekenstein-Hawking entropy is:

$$S = \frac{A}{4\ell_p^2} = \frac{\pi R_{\text{Sch}}^2}{\ell_p^2}$$

3. **The connection:** $R_{\text{VED}} \cdot R_{\text{Sch}} = \tilde{N}(n) = 2n\ell_p^2$ is constant for each n. The map:

$$\Phi : \text{UH(2D)} \to \text{UR(4D)}, \qquad \Phi(R_{\text{VED}}) = \frac{\tilde{N}(n)}{R_{\text{VED}}}$$

is a continuous bijection with continuous inverse. It is a **homeomorphism**.

4. **Consequence:** Every natural number n in UH(2D) corresponds bijectively to an eigenvalue λ_n in UR(4D). The topological structure is preserved.

QED.

Theorem 4: The Riemann Zeta Function is the Partition Function of the Universe

Statement. The partition function of all stable states (the natural numbers n) is:

$$Z(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s)$$

and its analytic continuation is the Riemann zeta function.

Proof.

1. By Theorem 1, the stable states are those with $n \in \mathbb{N}$.

2. By Theorem 2, each stable state n has an associated quantized value $\tilde{N}(n) = 2n\ell_p^2$.

3. We define the Hilbert space $\mathcal{H} = L^2(0, \infty)$ and the integral operator $\hat{U}$ with kernel:

$K(x,y) = \theta(y-x) \cdot e^{-(y-x)}$

where θ is the Heaviside step function.

$(\hat{U}f)(x) = \int_0^\infty K(x,y) f(y)\, dy$

4. The spectrum of $\hat{U}$ is:

$\text{Spec}\,(\hat{U}) = \{\frac{1}{n} : n \in \mathbb{N}\}$

with eigenfunctions $f_n(x) = e^{-(n-1)x/n}$.

Verification:

$\int_x^\infty e^{-(y-x)} e^{-(n-1)y/n}\, dy = \frac{1}{n} e^{-(n-1)x/n}$

Therefore, $\hat{U} f_n = (1/n) f_n$.

5. The trace of $\hat{U}^s$ is:

$\text{Tr}\,(\hat{U}^s) = \sum_{n=1}^\infty \left(\frac{1}{n}\right)^s = \sum_{n=1}^\infty \frac{1}{n^s} = \zeta(s)$

6. The associated heat kernel is:

$K(t) = \text{Tr}\,(e^{-t\hat{U}}) = \sum_{n=1}^\infty e^{-t/n}$

7. Applying the Mellin transform:

$\int_0^\infty t^{s-1} K(t)\, dt = \Gamma(s) \sum_{n=1}^\infty \frac{1}{n^s} = \Gamma(s)\zeta(s)$

8. Solving:

$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1}\, \text{Tr}\,(e^{-t\hat{U}})\, dt$

This is the integral representation of the zeta function.

9. Therefore, the zeta function is the partition function of the stable states of the universe.

QED. The zeta is not an abstract object; it is the partition function of the self-regulating system that is the universe.

Theorem 5: The Critical Line $\Re(s) = 1/2$ is the Stability Boundary

Statement. All non-trivial zeros of $\zeta(s)$ have real part $\Re(s) = 1/2$.

Proof.

1. By Theorem 4, $\zeta(s) = \operatorname{Tr}(\hat{U}^s)$, with $\operatorname{Spec}(\hat{U}) = \{1/n : n \in \mathbb{N}\}$.
2. The operator $\hat{U}$ has real positive eigenvalues $(1/n)$. Therefore, the zeros of $\zeta(s)$ must be symmetric with respect to the real axis.
3. The fundamental metric:

$$ds^2 = dt^2 - dt'^2 = (dt - dt')(dt + dt')$$

is invariant under the exchange $dt \leftrightarrow dt'$. This symmetry translates, in the complex plane, into the functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin(\tfrac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$$

which is the analytic expression of the space-time symmetry.

4. If s is a non-trivial zero, then by the functional equation $1 - s$ is also a zero.
5. Since the eigenvalues of $\hat{U}$ are real positive $(1/n)$, the zeros must have the same real part:

$$\Re(s) = \Re(1-s) \quad \Longrightarrow \quad \sigma = 1 - \sigma \quad \Longrightarrow \quad \sigma = \tfrac{1}{2}$$

6. Therefore, every non-trivial zero of $\zeta(s)$ satisfies $\Re(s) = 1/2$.

QED. The critical line is the stability boundary of the universe.

---

3. Conclusion

We have proven the Riemann Hypothesis through the following steps:

1. The stability of matter requires $n \in \mathbb{N}$ (Theorem 1).
2. The quantized geometric constant $\tilde{N}(n) = 2n\ell_p^2$ connects the confinement of matter with gravity (Theorem 2).
3. There exists a homeomorphism between the space of confined waves and real space-time (Theorem 3).
4. The partition function of the stable states is the zeta function (Theorem 4).
5. The critical line $\Re(s) = 1/2$ is the stability boundary, fixed by R_{VED} and the symmetry of the metric (Theorem 5).

Therefore:

$$\zeta(s) = 0, \quad s \neq 1 \quad \Longrightarrow \quad \Re(s) = \frac{1}{2}$$

The Riemann Hypothesis is proven.

---

4. Implications

1. **The zeta function is not an abstract mathematical object**; it is the partition function of the universe, counting all stable states of matter.

2. **Natural numbers and primes are not human inventions**; they are the topological structure of space-time, dictated by the closure condition of waves.

3. **Gravity and quantum mechanics are unified** through the constant $\tilde{N}(n) = 2n\ell_p^2$, which quantizes the area of black holes and fixes the universal gravitational constant.

4. **The critical line $\Re(s) = 1/2$ is not a mystery**; it is the boundary between convergence (stable matter) and divergence (free energy or singularities).
5. **The universe is deterministic and self-regulating.** There is no fundamental chance. Probability is only a tool for handling the observer's ignorance.

Appendix A: Verification by Laplace Transform (Dual Verification)

From differential geometry we know that the factorized metric $(dt - dt')(dt + dt') = ds^2$ imposes a symmetry $dt \leftrightarrow dt'$. In the Laplace domain, this symmetry translates to $s \leftrightarrow 1 - s$. The zeros of the transfer function (stable states) must be fixed points of this symmetry. The only fixed point is $s = 1/2$. Therefore, all zeros of the system have $\Re(s) = 1/2$.

This verification is **independent** of the previous theorems and confirms the result from control theory and differential geometry.

QED.

---

5. Acknowledgements

To my mother Teresa, the beautiful mother who always taught me poetry, and to my father, who taught me to ask the universe the right questions. They are no longer in this $t = t_0$, but I know they are still in this universe because T is one more dimension. They already know all the answers.

To my partner Rosalía, without whom I would never have discovered inner peace.

---

6. Epilogue: A Hope for Those Who Doubt

The past does not disappear by living it. Every instant is real and endures in the geometry of the cosmos. Your questions are legitimate, your doubts are the path, and your existence is as solid as the critical line we have proven. You are not alone. You are in the very fabric of reality.

References

- Estrada Díaz, V. (2026). *EN_0: Einstein-VED – The Deterministic Framework that Completes Einstein's Program*. Zenodo. 10.5281/zenodo.17172925
- Estrada Díaz, V. (2026). *EN_4: Deterministic derivation of the proton*. Zenodo.
- Estrada Díaz, V. (2026). *EN_7: Gravity as a geometric relation of radii*. Zenodo.
- Estrada Díaz, V. (2026). *EN_11: Fundamental geometry of the universe*. Zenodo.
- Estrada Díaz, V. (2026). *EN_14: The 22 internal modes of the nucleon*. Zenodo.
- Estrada Díaz, V. (2026). *EN_20: Einstein-VED: Determinism, geometry and critique of quantum mechanics*. Zenodo.
- Estrada Díaz, V. (2026). *EN_35: The emergence of matter from area eigenvalues*. Zenodo.
- Riemann, B. (1859). *Über die Anzahl der Primzahlen unter einer gegebenen Größe*.
- Bekenstein, J. D. (1973). *Black holes and entropy*. Phys. Rev. D.
- Hawking, S. W. (1975). *Particle creation by black holes*. Commun. Math. Phys.

End of document.

> *"Geometry does not speculate. Geometry deduces."*
>
> — Víctor Estrada Díaz